Genetic Algorithms

Dr. Mahmoud Nabil
mnmahmoud@ncat.edu

North Carolina A & T State University

September 6, 2021

Agenda

1. Counting
   - Product Rules
   - Permutation
   - Combination
2. Probability Theory
   - Basics
   - More on Probability

Outline

Outline

Counting

Counting of exact quantities of patterns, classifications, or distinct grouping fall under Combinatorics or Combinatorial analysis.

Counting Principle

With two experiments M (with m outcomes) and N (with n outcomes), there are m . n total possible outcomes of the compound experiment MN

- Also known as product rule
- Can be proved using matrix form and the cartesian product between sets, however the following illustrations.

Example

A student is certain he will get A or a B in Data Structures. He is not sure whether he will get A, B, C, D, or F in Genetic Algorithms 303. How many different grading possibilities are there.

Example

A student is certain he will get A or a B in Data Structures. He is not sure whether he will get A, B, C, D, or F in Genetic Algorithms 303. How many different grading possibilities are there.

Ans.

There are $m.n = 2.5 = 10$ possibilities.

AA, AB, AC, AD, AF, BA, BB, BC, BD, BF

Example

How many unique license plates can be constructed where the first three characters are letters of the alphabet and the last three characters are decimal digits?

Example

How many unique license plates can be constructed where the first three characters are letters of the alphabet and the last three characters are decimal digits?

Ans.

There are $26.26.26.10.10.10 = 17{,}576{,}000$ license plates.

Example

How many unique license plates can be constructed using the coding scheme of Example 2 when no repetition is allowed among the letters or the digits.

Example

How many unique license plates can be constructed using the coding scheme of Example 2 when no repetition is allowed among the letters or the digits.
Ans.
There are $26.25.24.10.9.8 = 11{,}232{,}000$ license plates.

Outline

Permutation

Permutation

A permutation is an ordered arrangement of a set of different items.

Ex.

Consider arranging the three letters A, B, C
We enumerate the result as ABC, ACB, BAC, BCA, CAB, CBA.

number of permutations of n objects $= n.(n-1).(n-2)...3.2.1 = n!$

Example

How many batting orders are there on a nine person baseball team.

Example

How many batting orders are there on a nine person baseball team. **Ans.** There are $9! = 9.8.7...3.2.1$=362,880

Example

Suppose you have 4 papers on genetic algorithms, 6 papers on neural networks, and 7 on fuzzy logic. How many arrangement are there if each classification is always grouped together.

Example

Suppose you have 4 papers on genetic algorithms, 6 papers on neural networks, and 7 on fuzzy logic. How many arrangement are there if each classification is always grouped together.

Ans.

There are 4!.6!.7!.3! possible arrangements.

Partial Ordering

Sometimes we are interested in the total numbering of unique ordering of r objects chosen from a set of n objects.

$$P(n,r) = n.(n-1).(n-2)\dots(n-r+1)$$
$$P(n,r) = \frac{n!}{(n-r)!}$$

r-Permutation

How many nine person batting orders are possible on a 15 person baseball team, assuming every player can play every position?

r-Permutation

How many nine person batting orders are possible on a 15 person baseball team, assuming every player can play every position?

Ans.

There are $P(15, 9) = \frac{15!}{(15-9)!} = 1,816,214,400$

Outline

Combination

Suppose we are selecting two letters from the following set where the order within a selection matter {A,B,C}

Ans.

AB, AC, BA, BC, CA, CB

$$P(3,2) = \frac{3!}{(3-2)!} = 6$$

Combination

Suppose we are selecting two letters from the following set where the order within a selection matter {A,B,C}
Ans.

$$\text{AB, AC, BA, BC, CA, CB}$$
$$P(3,2) = \frac{3!}{(3-2)!} = 6$$

Combination Sometimes we are interested in the number of unique grouping of objects irrespective of their ordering.

$$\text{AB} = \text{BA, AC}=\text{CA, and BC}=\text{CB}$$

Thus, we divide by the count of different ordering of 2 objects and we get $\frac{3!}{2!.(3-2)!} = 3$

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$

Examples

- The U.S. senate contains 100 senators. How many five-member subcommittees may be formed?

Examples

- The U.S. senate contains 100 senators. How many five-member subcommittees may be formed?
  - $C(100, 5) = 75,287,520$

Examples

- The U.S. senate contains 100 senators. How many five-member subcommittees may be formed?
  - $C(100,5) = 75,287,520$
- In five-card draw poker, each player is dealt five cards face down. How many unique deals are there

Examples

- The U.S. senate contains 100 senators. How many five-member subcommittees may be formed?
    - $C(100, 5) = 75,287,520$
- In five-card draw poker, each player is dealt five cards face down. How many unique deals are there
    - $C(52, 5) = 2,598,960$

Examples

- The U.S. senate contains 100 senators. How many five-member subcommittees may be formed?
  - $C(100, 5) = 75, 287, 520$
- In five-card draw poker, each player is dealt five cards face down. How many unique deals are there
  - $C(52, 5) = 2, 598, 960$

Examples

- The U.S. senate contains 100 senators. How many five-member subcommittees may be formed?
  - $C(100, 5) = 75,287,520$
- In five-card draw poker, each player is dealt five cards face down. How many unique deals are there
  - $C(52, 5) = 2,598,960$

Outline

Outline

Events and Spaces

Space (S)

It is the set of all possible outcomes for an experiment.

Ex.

- Flip a single coin S= {Heads, Tails}
- Roll of a single coin S={1,2,3,4,5,6}
- Flip of two coins S={HH,HT,TH,TT}

Events and Spaces

Space (S)

It is the set of all possible outcomes for an experiment.

Ex.

- Flip a single coin S= {Heads, Tails}
- Roll of a single coin S={1,2,3,4,5,6}
- Flip of two coins S={HH,HT,TH,TT}

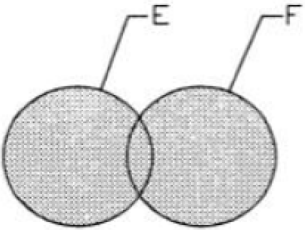
Event (E)

It is a subset of a possible outcomes for an experiment.

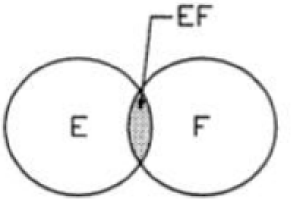
- At least one head in two tosses E= {HH, HT, TH}
- Roll of a die with a value greater than three E={4,5,6}

Events Operations

Union and Intersection:



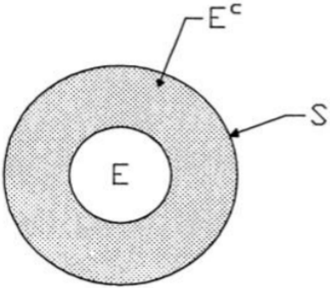
$E \cup F$



$E \cap F$

Events Operations

$E \cap F = \phi$



complement of an event E^c

Axioms of Probability

Three main axioms in the probability theory from which all other results may be derived.

Axiom 1

The probability of an event E must be between 0 and 1.

$$0 \leq P(E) \leq 1$$

Axioms of Probability

Three main axioms in the probability theory from which all other results may be derived.

Axiom 1

The probability of an event E must be between 0 and 1.

$$0 \leq P(E) \leq 1$$

Axiom 2

The probability of the space S must equal 1.

$$P(S) = 1$$

Axioms of Probability

Axiom 3

For any sequence of mutually exclusive events $E_i \quad i = 1, 2, \ldots, k$ such that $E_i \cap E_j = \phi$ for $i \neq j$

$$P(\cup_{i=1}^{i=k}) = \sum_{i=1}^{i=k} P(E_i)$$

The probability of the union of these events is the sum of their probabilities.

Consequences

Probability of the complementary event

$$P(E^c) = 1 - P(E)$$

Probability of the union of two events

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Equally Likely Outcomes

If all outcomes in a space S are equally likely, then the calculation of an Event E's probability is

$$P(E) = \frac{\text{number of points in event } E}{\text{number of points in space } S}$$

EX.

- The flip of an unbiased coin
- spin of a well-balanced roulette wheel
- roll of unweighted die.

Examples

- What is the probability rolling a 1 or a 2 on a fair die?

Examples

- What is the probability rolling a 1 or a 2 on a fair die?
  - $P(E) = \frac{2}{6} = \frac{1}{3}$

Examples

- What is the probability rolling a 1 or a 2 on a fair die?
  - $P(E) = \frac{2}{6} = \frac{1}{3}$
- What is the probability rolling an 8 on a pair of dice?

Examples

- What is the probability rolling a 1 or a 2 on a fair die?
  - $P(E) = \frac{2}{6} = \frac{1}{3}$
- What is the probability rolling an 8 on a pair of dice?
  - $P(E) = \frac{5}{36}$

Examples

- What is the probability rolling a 1 or a 2 on a fair die?
  - $P(E) = \frac{2}{6} = \frac{1}{3}$
- What is the probability rolling an 8 on a pair of dice?
  - $P(E) = \frac{5}{36}$
- What is the probability that a head appear at least once in 10 tosses of a fair coin?

Examples

- What is the probability rolling a 1 or a 2 on a fair die?
  - $P(E) = \frac{2}{6} = \frac{1}{3}$
- What is the probability rolling an 8 on a pair of dice?
  - $P(E) = \frac{5}{36}$
- What is the probability that a head appear at least once in 10 tosses of a fair coin?
  - $P(E) = 1 - \frac{1}{2}^{10}$

Examples

- What is the probability of being dealt a royal flush $\{A, J, Q, K, 10\}$ in five-card draw poker?

Examples

- What is the probability of being dealt a royal flush {A,J,Q,K,10} in five-card draw poker?
  - $P(E) = 4/C(52,5)$

Examples

- What is the probability of being dealt a royal flush {A,J,Q,K,10} in five-card draw poker?
  - $P(E) = 4/C(52,5)$
- What is the probability of being dealt a straight In five-card draw poker? A straight consists of five cards in order where all live cards are not of the same suit. (e.g., A 2 3 4 5 as long as all are from different suits)

Examples

- What is the probability of being dealt a royal flush {A,J,Q,K,10} in five-card draw poker?
  - $P(E) = 4/C(52,5)$
- What is the probability of being dealt a straight In five-card draw poker? A straight consists of five cards in order where all live cards are not of the same suit. (e.g., A 2 3 4 5 as long as all are from different suits)
  - $10.(4^5 - 4)/C(52,5)$

Outline

Conditional Probability

- In many real life cases, one event depend on another.
  - Probability to get an A depend on your semester work
- Conditional probability; symbolically we write $P(E|F)$. The probability of event E given event F has occurred.
- The intersection rule can be defined as

$$P(E \cap F) = P(F).P(E|F) = P(E).P(F|E)$$

Example

Joyce has a choice between two courses, one in genetic algorithms and one in fluid mechanics. If she has a 50 percent chance of receiving an 'A' In the genetic algorithms course and a 75 percent chance of getting an 'A' in the fluid mechanics course, what are her chance of getting an 'A' and takes the genetic algorithms course if she decide between the two courses on the toss of a fair coin?

Example

Joyce has a choice between two courses, one in genetic algorithms and one in fluid mechanics. If she has a 50 percent chance of receiving an 'A' In the genetic algorithms course and a 75 percent chance of getting an 'A' in the fluid mechanics course, what are her chance of getting an 'A' and takes the genetic algorithms course if she decide between the two courses on the toss of a fair coin?

Ans.

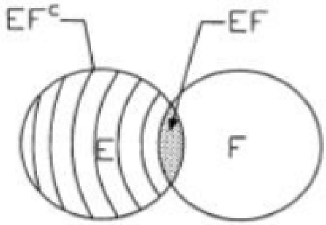
Let A be the event where Joyce receives an A, and let G be the event where she take the GA course.

$P(A \cap G) = P(G)P(A|G) = 0.5 \times 0.5 = 0.25$

Partitions of an Event

- It should be noted that the event E can be partitioned as $E \cap F^c$ and $E \cap F$
- Thus, $P(E) = P(E \cap F^c) + P(E \cap F)$
- Using the conditional probability

$$P(E) = P(F)P(E|F) + P(F^c)P(E|F^c)$$



Example

In the previous Example, suppose that Joyce can take fluid mechanic (event F) or genetic algorithm (event G) but not both, and again suppose that she makes her decision with the unbiased coin to . Calculate the probability of her making an A (event A).

Example

In the previous Example, suppose that Joyce can take fluid mechanic (event F) or genetic algorithm (event G) but not both, and again suppose that she makes her decision with the unbiased coin to . Calculate the probability of her making an A (event A).

Ans. Partition the A event on the mutually exclusive events G and F:
$P(A) = P(A|G)P(G) + P(A|F)P(F), = 0.5(0.5) + 0.75(0.5) = 0.625$

Bayes' Rule

It describes the probability of an event, based on prior knowledge of conditions that might be related to the event.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F|E)P(E)}{P(F)} = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}$$

Independent Events

Two events E and Fare said to be independent when the conditional probability $P(E|F)$ is equal to $P(E)$ alone. Thus, for independent events.

$$P(E \cap F) = P(E)P(F)$$

- What is the probability of rolling a deuce on a pair of dice?

Independent Events

Two events E and Fare said to be independent when the conditional probability $P(E|F)$ is equal to $P(E)$ alone. Thus, for independent events.

$$P(E \cap F) = P(E)P(F)$$

- What is the probability of rolling a deuce on a pair of dice?
  - $P(\text{toss}_1 = 1 \cap \text{toss}_2 = 1) = P(\text{toss}_1 = 1).P(\text{toss}_2 = 1) = 1/36$

Independent Events

Two events E and Fare said to be independent when the conditional probability $P(E|F)$ is equal to $P(E)$ alone. Thus, for independent events.

$$P(E \cap F) = P(E)P(F)$$

- What is the probability of rolling a deuce on a pair of dice?
  - $P(\text{toss}_1 = 1 \cap \text{toss}_2 = 1) = P(\text{toss}_1 = 1).P(\text{toss}_2 = 1) = 1/36$
- What is the probability n heads in n fair coin tosses?

Independent Events

Two events E and Fare said to be independent when the conditional probability $P(E|F)$ is equal to $P(E)$ alone. Thus, for independent events.

$$P(E \cap F) = P(E)P(F)$$

- What is the probability of rolling a deuce on a pair of dice?
  - $P(\text{toss}_1 = 1 \cap \text{toss}_2 = 1) = P(\text{toss}_1 = 1).P(\text{toss}_2 = 1) = 1/36$
- What is the probability n heads in n fair coin tosses?
  - $\frac{1}{2}^n$

Binomial Distribution

- If we perform a sequence of trials where each trial has a constant probability of success, P(success) = p.

Binomial Distribution

- If we perform a sequence of trials where each trial has a constant probability of success, $P(\text{success}) = p$.
- Then, The single experiment is called a Bernoulli trial and clearly the two possible outcomes, success and failure.

Binomial Distribution

- If we perform a sequence of trials where each trial has a constant probability of success, P(success) = p.
- Then, The single experiment is called a Bernoulli trial and clearly the two possible outcomes, success and failure.
- The probability of exactly k successes in n Bernoulli trials can be calculated (assuming independence of the trials) as follows

$$P(k \text{ success in } n \text{ trials}) = C(n,k)p^k(1-p)^{n-k}$$

Binomial Distribution

- If we perform a sequence of trials where each trial has a constant probability of success, P(success) = p.
- Then, The single experiment is called a Bernoulli trial and clearly the two possible outcomes, success and failure.
- The probability of exactly k successes in n Bernoulli trials can be calculated (assuming independence of the trials) as follows

$$P(k \text{ success in } n \text{ trials}) = C(n,k)p^k(1-p)^{n-k}$$

- This computation is true because a particular sequence of k successes requires exactly k successes and $n-k$ failures.

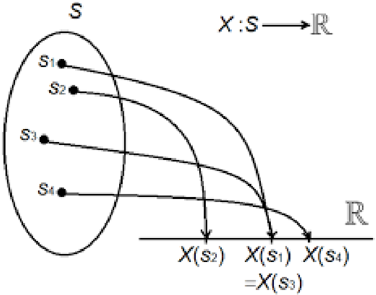
Binomial Distribution

- If we perform a sequence of trials where each trial has a constant probability of success, P(success) = p.
- Then, The single experiment is called a Bernoulli trial and clearly the two possible outcomes, success and failure.
- The probability of exactly k successes in n Bernoulli trials can be calculated (assuming independence of the trials) as follows
$$P(k \text{ success in } n \text{ trials}) = C(n, k)p^k(1-p)^{n-k}$$
- This computation is true because a particular sequence of k successes requires exactly k successes and $n-k$ failures.
- This probability distribution is called a binomial probability distribution

Expected Value of Random Variable

- Assume we want to calculate the usual outcome of some trial or trials of a random process.

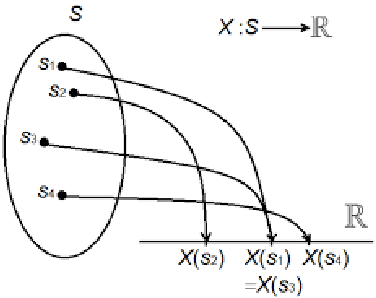
Random Variable



Expected Value of Random Variable

- Assume we want to calculate the usual outcome of some trial or trials of a random process.
- This is known as the expected value of a random variable.

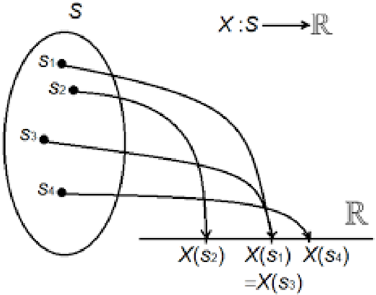
Random Variable



Expected Value of Random Variable

- Assume we want to calculate the usual outcome of some trial or trials of a random process.
- This is known as the expected value of a random variable.
- The expected value of a discrete random variable x is defined as $E(x) = \sum_{i=1}^{i=n} x.P(x)$

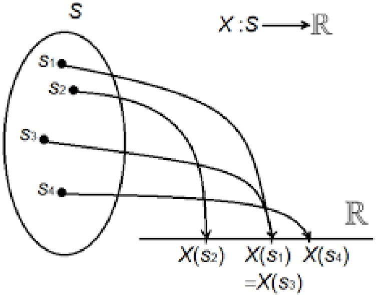
Random Variable



Expected Value of Random Variable

- Assume we want to calculate the usual outcome of some trial or trials of a random process.
- This is known as the expected value of a random variable.
- The expected value of a discrete random variable x is defined as $E(x) = \sum_{i=1}^{i=n} x.P(x)$
- We may also be interested in the expected value of some function of a random variable. $E(g(x)) = \sum_{i=1}^{i=n} g(x).P(x)$

Random Variable



Example

A gambler pays \$4.00 to roll a single die where he receives the face value in return (\$1.00 for an ace, \$2.00 for a deuce, etc.). What are his expected net winnings (losses)?

Example

A gambler pays \$4.00 to roll a single die where he receives the face value in return (\$1.00 for an ace, \$2.00 for a deuce, etc.). What are his expected net winnings (losses)?

Ans.

$E(\text{gross\_return}) = 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6 = \$3.5$

Net expected return is = -\$4.00 + \$3.50 = -\$0.50.

Limit Theorem

Theorem (Strong law of large numbers)

Assume a sequence of independent, identically distributed random variables x_n $i = 1, 2, \dots, n$ with finite expected value. With probability 1:

$$\frac{x_1+x_2+\cdots+x_n}{n} \to E(x) \text{ as } n \to \infty$$

References

- Goldenberg, D.E., 1989. Genetic algorithms in search, optimization and machine learning.
- Michalewicz, Z., 2013. Genetic algorithms + data structures= evolution programs. Springer Science & Business Media.

Questions